Name: ____________________________________

# Math 309 Practice Final 2

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**Instructions**: You may use a calculator for this exam. Please turn off all cell phones and pagers. You must show all work. Wherever a general solution is required, the solution must be in explicit form.

1. (25 points) Compute a sine Fourier series for the function

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1/2 \\ 1 - x, & 1/2 \leq x \leq 1. \end{cases}$$

2. (25 points) A one meter long elastic string is fixed at each end and its position $u(x,t)$ satisfies the PDE

$$u_{tt} = u_{xx}.$$

The string is plucked so that its initial position is

$$u(x,0) = \begin{cases} x, & 0 \le x \le 1/2 \\ 1-x, & 1/2 \le x \le 1. \end{cases}$$

Noting that this is the function from problem 1, solve for the function $u(x,t)$ with $0 \le x \le 1$ and $t \ge 0$ in terms of the Fourier series computed in problem 1.

3. (20 points) Find the general solution to the system of differential equations:

$$\mathbf{x}' = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} \sin x \\ 0 \end{pmatrix}.$$

4. (20 points) With

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix},$$

compute $e^A$. You may express $e^A$ as a product of matrices and their inverses. In other words, don't actually compute the inverse of $M$, just write $M^{-1}$.

5. (20 points) Solve the following initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$